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Osaka University

1120. 河口空間ノ共形幾何學

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§1. 次数 M の河口空間即ち曲線 $x^i(t)$ ($i=1, 2, \dots, n$) , $t_0 \leq t \leq t_1$ 間ノ弧ノ長ヲ積分

$$(1.1) \quad S = \int_{t_0}^{t_1} F(x, x^{(1)}, \dots, x^{(M)}) dt$$

ヲ定義サレル空間ヲ考ヘル。ユラデ (1.1) が幾何學的意味ヲモツタ $x = \lambda$, (1.1) ノ形ガ媒介変数 t ノスベテノ変換

$$(1.2) \quad \bar{t} = \bar{t}(t)$$

ニ對シテ不変デナケレバナラナイ。 (1.1) = 對應シテ次数 M ノ線素 $(x^{(1)}, x^{(2)}, \dots, x^{(M)})$ ノウケル変換

$$x^{i(\bar{\mu})} = \sum_{\nu=1}^{\mu} a_{\nu}^{\mu} x^{i(\nu)}$$

ハ M -次元変換群ヲナシ、無限小演子 $\Delta_1, \dots, \Delta_M$ ハ

$$(1.3) \quad \Delta_K = \sum_{\lambda \geq K} \binom{\lambda}{K} x^{(\lambda-K+1)i} \frac{\partial}{\partial x^{(\lambda)i}}$$

デ與ヘラレル。 Δ_K, Δ_H , Kommutator $[\Delta_K, \Delta_H]$ ハ

$$(1.4) \quad [\Delta_K \Delta_H] = \frac{(K-H)(K+H-1)!}{K! H!} \Delta_{K+H-1}$$

トナル。之ヲ用ヒレバ、(1.1) が媒介変数ノ変換(1.2) ア不
変ナルタメノ條件、即チ與ヘラレタ スカラー F が (1.2)
ヲ

$$\bar{F}(x, x^{(1)}, \dots, x^{(M)}) = \frac{dt}{dt} F(x, x^{(1)}, \dots, x^{(M)})$$

1 如ク変換スルタメノ條件ハ次ノ形ニカケユトが出来ル。

$$(1.5) \quad \Delta_1 F = F, \quad \Delta_K F = 0, \quad (2 \leq K \leq M)$$

$p(x, x^{(1)}, \dots, x^{(M)})$ ノ座標 (x) 及ビ α 次マデノ線
素 = depend スル任意ノ恒等的ニ零デ + イ スカラート
スル。即チ

$$\Delta_K p = 0, \quad (1 \leq K \leq \alpha)$$

ヲ満足スル量トスルトキ、 F ノ変化

$$(1.6) \quad \bar{F} = p \cdot F$$

ヲ河川空間 F ノ次数 α ノ共形変換トイフ。ユエデ $0 \leq \alpha \leq M-1$ トスル。

任意ノ点デ互ニ α 次ノ接触ヲナス線素ノ曲線ノ弧ハ、
 α 次ノ共形変換デ *Streckentreu* = 寫像サレル。

(1.6) デ不変ノ量ヲ求メルタメニ スカラー F = 関シテ次
ノ假定ヲオク。

$$\Delta_M F = 0 \text{ カラ } F_{(M)i} x^{(1)i} = 0,$$

従ッテ $F_{(M)i} (M)_j x^{(1)i} = 0$ 従ッテ行列 $(F_{(M)i} (M)_j)$

階数の高々 $n-1$ デアル。我々ハ ソレガ丁度 $n-1$ = 等シ
イト假定スル。然レトキハ $F_{(M) i (M) j} =$ 關スル行列 $(F_{(M) i (M) j})$
ノ代数余因子ヲ H_i^{ij} トスルヤ

$$H_i^{ij} = F_i x^{(1) i} x^{(1) j}$$

トナルヤウナ函数 F_1 ヲ求ムルコトが出来ル。

F_1 ハ再ビ $F_{(M) i (M) j} x^{(1) i} = 0$ ナル關係ヲ満足スル。

行列 $(F_{(M) i (M) j})$ ノ階数ガ $n-1$ = 等シト假定シテ

$$\text{Adj } F_{(M) i (M) j} = H_i^{ij}$$

トオケバ $H_i^{ij} = F_2 x^{(1) i} x^{(1) j}$ トナル様ナ F_2 ガ求マル。

F_1, F_2 ハ共形変換, 媒介変數ノ変換又ビ座標変換デ次
ノ如ク変換スル。

$$\bar{F}_1 = \rho^{n-1} \left(\frac{d\bar{t}}{dt} \right)^{(2M-1)(n-1)+2} \left| \frac{\partial x}{\partial \bar{x}} \right|^2 F_1$$

$$\bar{F}_2 = \rho^{(n-1)^2} \left(\frac{d\bar{t}}{dt} \right)^{((2M-1)n+3)(n-1)+2} \left| \frac{\partial x}{\partial \bar{x}} \right|^{2n} F_2$$

$$\text{然ッテ } \bar{F}_2^{-1} \bar{F}_1^n = \rho^{n-1} \left(\frac{d\bar{t}}{dt} \right)^{n-1} F_2^{-1} F_1^n$$

$$\text{故ニ } \theta = \log n(F), F^{-1} |F_2^{-1} F_1^n|^{\frac{1}{n-1}}$$

デ定義サレル θ ハ intrinsic + スカラーデ且ツ共形不変
量デアアル。

$$\theta^{(1)} = \psi, \quad \chi = F/\psi$$

トオケバ ψ は共形不変量ヲ媒介変数ノ変換デ F ト同ジ変換ヲウケ、 κ は *intrinsic* デ共形変換デ

$$\bar{\kappa} = \rho \kappa$$

ノ如ク変換スル。

$$\text{定理 2. (1.9)} \quad \bar{E}_{i\mu} = \sum_{\lambda=\mu}^M \binom{\lambda}{\mu} \left(\frac{1}{F}\right)^{(\lambda-\mu)} E_{i\lambda},$$

$$(\mu = \alpha+1, \dots, M)$$

ナル $M-\alpha$ ヲ共変ベクトルハ、次数 μ ノ共形変換ニ對シテ不変デアル。^{*}

証明. $\bar{F} = \rho F$ カラ

$$\log \bar{F} = \log \rho + \log F$$

從ツテ $\mu \geq \alpha+1$ ナラバ

$$\sum_{\lambda=\mu}^M (-1)^\lambda \binom{\lambda}{\mu} (\log \bar{F})_{(\lambda-\mu)i} = \sum_{\lambda=\mu}^M (-1)^\lambda \binom{\lambda}{\mu} (\log F)_{(\lambda-\mu)i}$$

シカバ

$$\begin{aligned} \sum_{\lambda=\mu}^M (-1)^\lambda \binom{\lambda}{\mu} (\log F)_{(\lambda-\mu)i} &= \sum_{\lambda=\mu}^M (-1)^\lambda \binom{\lambda}{\mu} \left(\frac{F_{(\lambda-\mu)i}}{F} \right)^{(\lambda-\mu)} \\ &= \sum_{\lambda=\mu}^M \sum_{\nu=0}^{\lambda-\mu} (-1)^\lambda \binom{\lambda}{\mu} \binom{\lambda-\mu}{\nu} \left(\frac{1}{F} \right)^{(\nu)} F_{(\lambda-\mu-\nu)i} \end{aligned}$$

$$* \quad E_{i\lambda} = \sum_{\mu=\lambda}^M (-1)^\mu \binom{\mu}{\lambda} (F_{(\mu-\lambda)i})^{(\mu-\lambda)} \quad \wedge -0 \leq \lambda \leq M = \text{對シ共}$$

変ベクトルノ成分ヲナシ *Synge* ノベクトルトヨバレル。

$$\begin{aligned}
&= \sum_{\lambda=\mu}^M \sum_{\nu=0}^{\lambda-\mu} -\binom{\mu+\nu}{\mu} \left(\frac{1}{F}\right)^{(\nu)} (-1)^\lambda \binom{\lambda}{\mu+\nu} F_{(\lambda)i}^{--(\lambda-\mu-\nu)} \\
&= \sum_{\alpha=\mu}^M \binom{\alpha}{\mu} \left(\frac{1}{F}\right)^{(\alpha-\mu)} \sum_{\lambda=\alpha}^M (-1)^\lambda \binom{\lambda}{\alpha} F_{(\lambda)i}^{--(\lambda-\alpha)} \\
&= \sum_{\lambda=\mu}^M \binom{\lambda}{\mu} \left(\frac{1}{F}\right)^{(\lambda-\mu)} E_{i\lambda}
\end{aligned}$$

即ち $t_{i\mu} = E_{i\mu}(\log F)$ $\bar{t}_{i\mu} = t_{i\mu} + \nu$.

$t_{i\mu+1}, \dots, t_{iM}$ は媒分変数, 変換で不変デハナク

$$\begin{aligned}
(1.10) \quad \Delta_K t_{i\mu} &= -\binom{\mu+K}{K} t_{i\mu+K-1} \quad 2 \leq K \leq M-\mu+1 \\
\Delta_1 t_{i\mu} &= -\mu t_{i\mu}.
\end{aligned}$$

ナル関係が成立スル, 即ち

$$\Delta_K E_{i\mu} = -\binom{\mu+K-1}{K} E_{i\mu+K-1} - \delta'_K E_{i\mu}$$

及ビ

$$\Delta_K \left(\frac{1}{F}\right)^{(H)} = \frac{H+1-2K}{H+1} \binom{H+1}{K} \left(\frac{1}{F}\right)^{(H-K+1)}$$

ナル関係ヲ用テバ

$$\begin{aligned}
\Delta_K t_{i\mu} &= \Delta_K \sum_{\nu=\mu}^M \binom{\nu}{\mu} \left(\frac{1}{F}\right)^{(\nu-\mu)} E_{i\nu} \\
&= \sum_{\nu \geq \mu} \left[\binom{\nu}{\mu} \frac{\nu-\mu-2K+1}{\nu-\mu+1} \binom{\nu-\mu+1}{K} \left(\frac{1}{F}\right)^{(\nu-\mu-K+1)} E_{i\nu} \right. \\
&\quad \left. - \binom{\nu}{\mu} \left(\frac{1}{F}\right)^{(\nu-\mu)} \binom{\nu+K-1}{K} E_{i\nu+K-1} \right]
\end{aligned}$$

$$= \sum_{\nu \geq \mu+K-1} \left[\binom{\nu}{\mu} \frac{\nu-\mu-2K+1}{\nu-\mu+1} \binom{\nu-\mu+1}{K} - \binom{\nu-K+1}{\mu} \binom{\nu}{K} \right] \left(\frac{1}{R}\right)^{(\nu-\mu-K+1)} E_{i\nu}$$

$$= - \binom{\mu+K}{K} \ell_{i, \mu+K-1}$$

$$\Delta_1 \ell_{i\mu} = -(\mu+1) \ell_{i, \mu} + \ell_{i\mu} = -\mu \ell_{i\mu}$$

§2. $\lambda = \rho$ 特殊函数に制限した場合を論じよう。
この場合 M は *Syngge* ベクトル、 M 個の線状結合トシテ、共形不変で且 *intrinsic* M 個の共変ベクトルヲ導クコトが出来る。

$E_{i\mu}$ は共形変換で λ 如ク変換スル。

$$(2.1) \quad \bar{E}_{i\mu} = c_{\mu}^{\lambda} E_{i\lambda}, \quad c_{\mu}^{\lambda} = \binom{\lambda}{\mu} \rho^{(\lambda-\mu)}$$

之ハ (1.9) を出シタト全く同じ計算ヲ証明スルコトが出来ル。

$$\bar{\kappa} = \rho \kappa \quad \text{カヲ}$$

$$\bar{\kappa}^{(\nu)} = \sum_{\mu=0}^{\nu} \binom{\nu}{\mu} \rho^{(\mu)} \kappa^{(\nu-\mu)}$$

従ッテ

$$(2.2) \quad \frac{\rho^{(\nu)}}{\rho} = \frac{\bar{\kappa}^{(\nu)}}{\bar{\kappa}} - \frac{\kappa^{(\nu)}}{\kappa} - \sum_{\mu=1}^{\nu-1} \binom{\nu}{\mu} \frac{\rho^{(\mu)}}{\rho} \frac{\kappa^{(\nu-\mu)}}{\kappa}$$

$$\geq 1 \quad \bar{E}_{i,1} = \sum_{0 \leq \nu \leq M-1} (\nu+1) E_{i, \nu+1} \rho^{(\nu)}$$

或ハ

$$\frac{\bar{E}_{i1}}{\bar{\kappa}} = \sum_{\nu=0}^{M-1} (\nu+1) \frac{E_{i\nu+1}}{\kappa} \frac{\rho^{(\nu)}}{\rho}$$

トカフ

$$\begin{aligned} \frac{\bar{E}_{i1}}{\bar{\kappa}} &= \sum_{\nu=0}^{M-2} (\nu+1) \frac{E_{i\nu+1}}{\kappa} \frac{\rho^{(\nu)}}{\rho} + \left[\frac{\bar{\kappa}^{(M-1)}}{\bar{\kappa}} - \frac{\kappa^{(M-1)}}{\kappa} \right. \\ &\quad \left. - \sum_{\nu=1}^{M-2} \binom{M-1}{\nu} \frac{\rho^{(\nu)}}{\rho} \frac{\kappa^{(M-\nu-1)}}{\kappa} \right] M \frac{E_{iM}}{\kappa} \\ &= \frac{E_{i1}}{\kappa} + \sum_{\nu=1}^{M-2} \frac{\rho^{(\nu)}}{\rho} \left[(\nu+1) \frac{E_{i\nu+1}}{\kappa} - \binom{M-1}{\nu} \frac{\kappa^{(M-\nu-1)}}{\kappa} M \frac{E_{iM}}{\kappa} \right] \\ &\quad + \left[\frac{\bar{\kappa}^{(M-1)}}{\bar{\kappa}} - \frac{\kappa^{(M-1)}}{\kappa} \right] M \frac{E_{iM}}{\kappa} \end{aligned}$$

從ツテ

$$\begin{aligned} (23) \quad \frac{\bar{E}_{i1}}{\bar{\kappa}} - \frac{\bar{\kappa}^{(M-1)}}{\bar{\kappa}} M \bar{C}_{iM} &= \frac{E_{i1}}{\kappa} - \frac{\kappa^{(M-1)}}{\kappa} M C_{iM} \\ &\quad + \sum_{\nu=1}^{M-2} \frac{\rho^{(\nu)}}{\rho} \left[(\nu+1) \frac{E_{i\nu+1}}{\kappa} - \binom{M-1}{\nu} \frac{\kappa^{(M-\nu-1)}}{\kappa} M \bar{C}_{iM} \right] \\ &\quad + (M-1) \bar{C}_{iM-1} \frac{\rho^{(M-2)}}{\rho} \end{aligned}$$

$$\bar{C}_{iM} = \frac{E_{iM}}{\kappa}, \quad C_{iM-1} = \frac{E_{iM-1}}{\kappa} - \frac{\kappa^{(1)}}{\kappa} M \bar{C}_{iM}$$

$$E_{iM} = (-1)^M F_{(M)i}, \quad \exists \nu \text{ 直} = \bar{C}_{iM} = C_{iM}$$

$$\text{又 } \bar{C}_{iM-1} = \bar{C}_{iM-1} \Rightarrow \rho \text{ 直} \Rightarrow \text{見 } \nu = \wedge \quad F = \left(\frac{1}{\rho}\right) \bar{F}, \quad \kappa = \left(\frac{1}{\rho}\right) \bar{\kappa}$$

トシテ上ト同ジ計算ヲ繰返セバ

$$\begin{aligned} \frac{E_{i1}}{\kappa} - \frac{\kappa^{(M-1)}}{\kappa} M \bar{C}_{iM} &= \frac{\bar{E}_{i1}}{\kappa} - \frac{\kappa^{(M-1)}}{\kappa} M \bar{C}_{iM} \\ &+ \Phi(E_{iM}, E_{iM-1}, \dots, \kappa, \kappa^{(1)}, \dots, \rho, \rho^{(1)}, \dots, \rho^{(M-3)}) \\ &- (M-1) \bar{C}_{iM-1} \frac{\rho^{(M-2)}}{\rho} \end{aligned}$$

ソノ式ト (2.3) トノ $\rho^{(M-2)}$ ノ含ム項及ビ $\rho, \rho^{(1)}, \dots$ ノ含マナイ項ヲ比較スレバ直ニ $\bar{C}_{iM-1} = C_{iM-1}$ ノ得ル。之ヲ繰返シテ行ケバ

$$\begin{aligned} (2.4) \quad \frac{\bar{E}_{i1}}{\kappa} - \sum_{v=1}^M \frac{(M-v+1) \kappa^{(M-v)}}{\kappa} \bar{C}_{iM-v+1} \\ = \frac{E_{i1}}{\kappa} - \sum_{v=1}^M \frac{(M-v+1) \kappa^{(M-v)}}{\kappa} C_{iM-v+1} \\ - \sum_{v=0}^{M-M-1} \frac{\rho^{(v)}}{\rho} \left[(v+1) \frac{E_{iv+1}}{\kappa} - \sum_{\alpha=0}^{M-1} \binom{M-\alpha-1}{v} \frac{\kappa^{(M-\alpha-v-1)}}{\kappa} (M-\alpha) C_{iM-\alpha} \right] \end{aligned}$$

$$(2.5) \quad C_{iM} = \frac{E_{iM}}{E\kappa}$$

$$C_{iM-1} = \frac{E_{iM-1}}{F\kappa} - \frac{\kappa^{(1)}}{\kappa} M C_{iM}$$

$$C_{iM-2} = \frac{E_{iM-2}}{\kappa} - \frac{\kappa^{(2)}}{\kappa} \binom{M}{2} C_{iM} - \frac{\kappa^{(1)}}{\kappa} \binom{M-1}{1} C_{iM-1}$$

$$C_{iM-M-1} = \frac{E_{iM-M-1}}{\kappa} - \frac{\kappa^{(M-1)}}{\kappa} \binom{M}{M-1} C_{iM}$$

$$-\frac{\kappa^{(M-2)}}{\kappa} \binom{M-1}{M-2} \bar{C}_{iM-1} \cdots - \frac{\kappa^{(1)}}{\kappa} \binom{M-\mu+2}{M-\mu+2} \bar{C}_{iM-\mu+2}$$

ヲ繰ル。即チ (2.4) ヲ假定スレバ

$$\begin{aligned} \frac{\bar{E}_{i1}}{\bar{\kappa}} - \sum_{\nu=1}^{\mu} \frac{(M-\nu+1)\bar{\kappa}^{(M-\nu)}}{\bar{\kappa}} \bar{C}_{iM-\nu+1} &= \frac{E_{i1}}{\kappa} - \sum_{\nu=1}^{\mu} \frac{(M-\nu+1)\kappa^{(M-\nu)}}{\kappa} C_{iM-\nu+1} \\ &+ \sum_{\nu=0}^{M-\mu-2} \frac{\rho^{(\nu)}}{\rho} \left[(\nu+1) \frac{E_{i\nu+1}}{\kappa} - \sum_{\lambda=0}^{M-1} \binom{M-\lambda-1}{\nu} \frac{\kappa^{(M-\lambda-\nu-1)}}{\kappa} (M-\lambda) C_{iM-\lambda} \right] \\ &+ \frac{\rho^{(M-\mu-1)}}{\rho} (M-\mu) \left[\frac{E_{iM-\mu}}{\kappa} - \sum_{\lambda=0}^{M-1} \binom{M-\lambda}{M-\mu} \frac{\kappa^{(M-\lambda)}}{\kappa} C_{iM-\lambda} \right] \\ \bar{C}_{iM-\mu} &= \frac{E_{iM-\mu}}{\kappa} - \sum_{\lambda=0}^{M-1} \binom{M-\lambda}{M-\mu} \frac{\kappa^{(M-\lambda)}}{\kappa} C_{iM-\lambda} \end{aligned}$$

トオケバ之ハ共形不変デ

$$\begin{aligned} \frac{\bar{E}_{i1}}{\bar{\kappa}} - \sum_{\nu=1}^{\mu} \frac{(M-\nu+1)\bar{\kappa}^{(M-\nu)}}{\bar{\kappa}} \bar{C}_{iM-\nu+1} &= \frac{E_{i1}}{\kappa} - \sum_{\nu=1}^{\mu} \frac{(M-\nu+1)\kappa^{(M-\nu)}}{\kappa} C_{iM-\nu+1} \\ &+ \sum_{\nu=0}^{M-\mu-2} \frac{\rho^{(\nu)}}{\rho} \left[(\nu+1) \frac{E_{i\nu+1}}{\kappa} - \sum_{\lambda=0}^{\mu} \frac{\kappa^{(M-\lambda-\nu-1)}}{\kappa} (M-\lambda) C_{iM-\lambda} \right] \\ &+ \left[\frac{\bar{\kappa}^{(M-\mu-1)}}{\bar{\kappa}} - \frac{\kappa^{(M-\mu-1)}}{\kappa} \right] (M-\mu) C_{iM-\mu} \end{aligned}$$

或ハ

$$\begin{aligned} \frac{\bar{E}_{i1}}{\bar{\kappa}} - \sum_{\nu=1}^{\mu+1} \frac{(M-\nu+1)\bar{\kappa}^{(M-\nu)}}{\bar{\kappa}} \bar{C}_{iM-\nu+1} &= \frac{E_{i1}}{\kappa} - \sum_{\nu=1}^{\mu+1} \frac{(M-\nu+1)\kappa^{(M-\nu)}}{\kappa} C_{iM-\nu+1} \\ &+ \sum_{\nu=0}^{M-\mu-2} \frac{\rho^{(\nu)}}{\rho} \left[(\nu+1) \frac{E_{i\nu+1}}{\kappa} - \sum_{\lambda=0}^{\mu+1} \binom{M-\lambda-1}{\nu} \frac{\kappa^{(M-\lambda-\nu-1)}}{\kappa} (M-\lambda) C_{iM-\lambda} \right] \end{aligned}$$

$$\text{定理3. } \bar{C}_{iM-\nu} = \frac{E_{iM-\nu}}{\kappa} - \sum_{\alpha=0}^{\nu-1} \binom{M-\alpha}{M-\nu} \frac{\kappa^{(\nu-\alpha)}}{\kappa} \bar{C}_{iM-\alpha}$$

$$\bar{C}_{iM} = \frac{E_{iM}}{\kappa}$$

ヲ定義サレル \$M\$ コノ共変ベクトル \$\bar{C}_{i1}, \dots, \bar{C}_{iM}\$ ハ共変不変量ナリ。

(2.5) ヲ書き直セバ

$$E_{iM} = \kappa \bar{C}_{iM}$$

$$E_{iM-1} = \kappa^{(1)} M \bar{C}_{iM} + \kappa \bar{C}_{iM-1}$$

$$E_{iM-2} = \kappa^{(2)} \binom{M}{2} \bar{C}_{iM} + \kappa^{(1)} \binom{M-1}{1} \bar{C}_{iM-1} + \kappa \bar{C}_{iM-2}$$

$$E_{iM-\nu} = \kappa^{(\nu)} \binom{M}{\nu} \bar{C}_{iM} + \kappa^{(\nu-1)} \binom{M-1}{\nu-1} \bar{C}_{iM-1}$$

$$+ \kappa^{(\nu-2)} \binom{M-2}{\nu-2} \bar{C}_{iM-2} + \dots + \kappa \bar{C}_{iM-\nu}$$

従ヒテ

$$\bar{C}_{iM-\nu} = \frac{1}{\kappa^{\nu+1}} \begin{vmatrix} \kappa & & & E_{iM} \\ M\kappa^{(1)} & \kappa & & E_{iM-1} \\ \binom{M}{2}\kappa^{(2)} & \binom{M-1}{1}\kappa^{(1)} & \kappa & E_{iM-2} \\ \dots & \dots & \dots & \dots \\ \binom{M}{\nu}\kappa^{(\nu)} & \binom{M-1}{\nu-1}\kappa^{(\nu-1)} & \binom{M-2}{\nu-2}\kappa^{(\nu-2)} & E_{iM-\nu} \end{vmatrix}$$

$$\bar{C}_{iM-\nu} = \sum_{\mu=0}^{\nu} C_{M-\nu}^{M-\mu} E_{iM-\mu}$$

$$C_{M-\nu}^{M-\mu} = \frac{1}{\kappa^{\nu+1}} \begin{vmatrix} \kappa & & & & 0 \\ M\kappa^{(1)} & & & & \vdots \\ \cdots & \cdots & \cdots & \cdots & \vdots \\ \binom{M}{\mu}\kappa^{(\mu)} & \binom{M-1}{\mu-1}\kappa^{(\mu-1)} & \cdots & \kappa & \cdots & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ \binom{M}{\nu}\kappa^{(\nu)} & \binom{M-1}{\nu-1}\kappa^{(\nu-1)} & \cdots & \cdots & \kappa^{(1)} & 0 \end{vmatrix}$$

$$= \binom{M-\mu}{M-\nu} \left(\frac{1}{\kappa} \right)^{(\nu-\mu)}$$

即ち

$$\text{定理 4. } \bar{C}_{i\mu} = \sum_{\lambda=\mu}^M C_{\mu}^{\lambda} E_{i\lambda}, \quad C_{\mu}^{\lambda} = \binom{\lambda}{\mu} \left(\frac{1}{\kappa} \right)^{(\lambda-\mu)}$$

$E_{i\mu}$ = 関係 τ の基本的 + 関係

$$E_{i\mu} x^{(1)i} = 0 \quad (2 \leq \mu \leq M), \quad E_{i1} x^{(1)i} = -F$$

が成立する。之レカラ

$$\bar{C}_{i\mu} x^{(1)i} = 0, \quad (2 \leq \mu \leq M), \quad \bar{C}_{i1} x^{(1)i} = -\frac{F}{\kappa}$$

$$\text{定理 5. } \xi_{i\lambda} = \frac{\kappa}{F} \sum_{\mu=\lambda}^M A_{\lambda}^{\mu} \bar{C}_{i\mu}$$

トオケル $\xi_{i\lambda}$ の *intrinsic* + 共変ベクトルデ共形変換 = ヨリ

$$\bar{\xi}_{i\lambda} = \rho^{\xi_{i\lambda}}$$

1 如ク変換サレル, $\rho = \kappa^{(\mu)i}$, $\bar{F} = \bar{F}(\rho)$ = 對スル変換ヲ

$$x^{(\bar{\mu})i} = \sum_{\lambda=1}^{\mu} a_{\lambda}^{\mu} \left(\frac{dt}{d\bar{t}}, \frac{d^2 t}{d\bar{t}^2}, \dots, \frac{d^{\mu-\lambda+1} t}{d\bar{t}^{\mu-\lambda+1}} \right) x^{(N)i}$$

$$\text{トシ } A_{\lambda}^{\mu} \wedge a_{\lambda}^{\mu}, \frac{dt}{d\bar{t}}, \frac{d^2 t}{d\bar{t}^2}, \dots, \text{所へ } \psi, \psi^{(1)} \dots \text{ヲ}$$

代入シタモ、

$$\text{即チ } A_{\lambda}^{\mu} = a_{\lambda}^{\mu} (\psi, \psi^{(1)}, \dots, \psi^{(\mu-\lambda)})$$

トスル。

証明. $E_{i\mu}, C_{\mu}^{\lambda}, A_{\lambda}^{\mu}$ へ $\bar{t} = \bar{t}(t)$ デ夫々次, 如キ変換ヲウケルコトカラ明カデアアル。^{*}

$$\bar{E}_{i\mu} = \frac{dt}{d\bar{t}} a_{\mu}^{\lambda} E_{i\lambda}$$

$$\bar{C}_{\mu}^{\lambda} = a_{\alpha}^{\lambda} \bar{a}_{\mu}^{\beta} C_{\beta}^{\alpha}$$

$$\bar{A}_{\lambda}^{\mu} = a_{\nu}^{\mu} A_{\lambda}^{\nu}$$

M-2 ヲ intrinsic + 共変ベクトル

$$E_{i\alpha+1}, \dots, E_{iM}$$

ハ次数 α の共形変換 = 對シ $\bar{E}_{i\mu} = \rho^{\alpha} E_{i\mu}$ 如ク変換サレル。

$E_{i\lambda}$ ハス $\bar{E}_{i\lambda}$ 線狀結合トシテ次, 形 = カケコト

* A. Kawaguchi, Differentialgeometrie höherer Ordnung. III

Jour Fac. Sci. Hokkaido Imp Univ (1941)

が出来る。

$$\varepsilon_{i\lambda} = \sum_{\mu=\lambda}^M K_{\lambda}^{\mu} \ell_{i\mu} \quad K_{\lambda}^{\mu} = \frac{\kappa}{\psi} \sum_{\alpha=\lambda}^{\mu} \binom{\mu}{\alpha} \psi^{(\mu-\alpha)} A_{\lambda}^{\mu}(\psi, \psi^{(1)}, \dots, \psi^{(M-\lambda)})$$

$$\begin{aligned} \text{即ち } \frac{F}{\kappa} \varepsilon_{i\lambda} &= \sum_{\lambda \leq \nu \leq M} \sum_{\mu=\lambda}^{\nu} A_{\lambda}^{\mu} \binom{\nu}{\mu} \left(\frac{1}{F}\right)^{(\nu-\mu)} E_{i\nu} \\ &= \sum_{\lambda \leq \mu \leq M} \sum_{\mu \leq \nu \leq M} A_{\lambda}^{\mu} \binom{\nu}{\mu} \left(\frac{\psi}{F}\right)^{(\nu-\mu)} E_{i\nu} \\ &= \sum_{\mu=\lambda}^M \sum_{\mu \leq \nu \leq M} \sum_{\alpha=0}^{\nu-\mu} \left(A_{\lambda}^{\mu} \binom{\nu}{\mu} \binom{\nu-\mu}{\alpha} \psi^{(\alpha)} \left(\frac{1}{F}\right)^{(\nu-\mu-\alpha)} \right) E_{i\nu} \\ &= \sum_{\mu=\lambda}^M \sum_{\mu \leq \nu \leq M} \sum_{\alpha=0}^{\nu-\mu} A_{\lambda}^{\mu} \binom{\mu+\alpha}{\mu} \psi^{(\alpha)} \binom{\nu}{\mu+\alpha} \left(\frac{1}{F}\right)^{(\nu-\mu-\alpha)} E_{i\nu} \\ &= \sum_{\alpha=\lambda}^M \left(\sum_{\mu=\lambda}^{\alpha} A_{\lambda}^{\mu} \binom{\alpha}{\mu} \psi^{(\alpha-\mu)} \right) \left(\frac{\psi}{\alpha}\right) \sum_{\nu=\alpha}^M \binom{\nu}{\alpha} \left(\frac{1}{F}\right)^{(\nu-\alpha)} E_{i\nu} \\ &= \sum_{\alpha=\lambda}^M K_{\lambda}^{\alpha} \ell_{i\alpha} \end{aligned}$$

$$\S 3. \quad g_{ij} = \psi^{2(M-1)} F F_{(M)i(M)j} + \varepsilon_{i1} \varepsilon_{j1}$$

トカケル g_{ij} は二階対称テンソルデヲノ行列式ハ一般
 $= 0$ デナリ,

$$g_{ij} x^{(1)i} = -F \varepsilon_{j1}, \quad g_{ij} x^{(1)i} x^{(1)j} = F^2$$

ナル關係が成立シ、且ツ共形変換デ

$$\bar{g}_{ij} = \rho^2 g_{ij}$$

1 如ク変換スル。然ツテ g_{ij} ヲ我々ノ空間ノ基本テンソルトシテ採用スルコトが出来ル。

X^j ヲ任意ノ *intrinsic* デ共形不変トベクトルトスレバ

$$\psi^{2(M-1)} F \cdot D_{jM-1} (F_{(M)i}) X^j = \psi^{2(M-1)} F \cdot F_{(M)i(M)j} \frac{dX^j}{dt} \\ + \frac{1}{M} \psi^{2(M-1)} F \left[F_{(M)i(M-1)j} + \psi^+ \cdot \psi^{(1)} F_{(M)i(M)j} \right] X^j$$

$$\kappa \varepsilon_{ij} \left\{ \left(\frac{\varepsilon_{ji}}{\kappa} X^j \right)^{(1)} \right\} = \varepsilon_{ij} \varepsilon_{ji} \frac{dX^j}{dt} + \kappa \varepsilon_{ij} \left[\left(\frac{\varepsilon_{ji}}{\kappa} \right)^{(1)} \right] X^j$$

ハ何レモ t ノ変換デ F ト同じ変換ヲスル反変ベクトルデアアル。之等カラ X^j ノ曲線 = 共形不変ト共変微分

$$\frac{\delta X^j}{dt} = \frac{dX^j}{dt} + \Gamma_i^j X^i$$

$$\Gamma_i^j = \frac{1}{M} \psi^{-(M-1)} F \cdot g^{ja} \left[F_{(M)a(M-1)i} + \psi^+ \psi^{(1)} F_{(M)a(M)i} \right] \\ - \frac{\kappa}{F} X^{(1)j} \left(\frac{\varepsilon_{ji}}{\kappa} \right)^{(1)}$$

ヲ定義スルコトが出来ル。

Γ_j^i ハ $2M+1$ 次ノ線素 = マデ *depend* スル函数デアアル。

$$D_p \Gamma_j^i = \sum_{\lambda \geq p} \binom{\lambda}{p} \Gamma_{j(\lambda-p)K}^i dX^{(\lambda-p)K} \quad (p=1, 2, \dots)$$

カヲ *intrinsic* テ共形不変ノ線型接続ノ度數

$$\sum_{0 \leq a \leq 2M} \Gamma_{jK}^{i(a)} dx^{(a)K}$$

ヲ導クコトが出来ル。之ヲ用ヒテ X^i ノ共変微分

$$\delta X^i = dX^i + \sum_{a=0}^{2M+1} \Gamma_{jK}^{i(a)} X^j dx^{(a)K}$$

ガ定義サレル。

共形不変ノ基本接続ノ決点, ρ ガ一般ノ函数ノ場合, 理論,
或ハ共変微分ノ次數ヲモ少シ低クスルコトハ出来ナイカ,
一点 $(x) =$ 於テ相隣ルニツノ線素 $(x^{(1)}, \dots, x^{(V)})$,
 $(x^{(1)} + dx^{(1)}, \dots, x^{(V)} + dx^{(V)})$ ノ間ノ角ヲ定義スルニハ
ドリスレバヨイカ等ノ問題が残ッテキルガ今回ハ之レヲ筆
ヲオク。